

TOPOLOGY II MIDTERM EXAM

Total Marks: 50

- (1) Let $f : S^1 \rightarrow S^1$ be a continuous map which is null homotopic. Show that f has a fixed point and f maps some point x to its antipode $-x$. (7 marks)
- (2) Let $f : S^2 \rightarrow S^2$ be a continuous map such that $f(x) \neq f(-x)$ for all $x \in S^2$. Show that f is onto. (7 marks)
- (3) Show that a topological space X is contractible, if and only if, X has the homotopy type of a one point space. (8 marks)
- (4) What is the fundamental group of the complement of a finite set of points in \mathbb{R}^3 ? Justify your answer. (7 marks)
- (5) Does there exist a retraction from the solid torus $B^2 \times S^1$ onto its boundary $S^1 \times S^1$? (7 marks)
- (6) Let X, Y, Z be path connected and locally path connected topological spaces. Let $q : X \rightarrow Y$, $r : Y \rightarrow Z$ be covering maps, let $p = rq$. Assume Z has a universal covering space. Then, show that $p : X \rightarrow Z$ is a covering map. (7 marks)
- (7) Draw a picture and describe a two fold path connected covering space of the figure eight. Is this covering space regular or not regular? (7 marks)